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Influence of Vacuum Energy on Scattering

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We deal with photon–electron scattering which occurs between two uncharged conducting parallel plates moving away from each other at a constant velocity. The electromagnetic vacuum field between two plates is defined by the configuration of space and also interacts with the electrons. We show the relevant operators for both the electron and photon fields and the computation of the corresponding Feynman propagator, S-matrix, and scattering cross section, taking into account the influence of the changeable vacuum field. Correction terms in the computed S-matrix and scattering cross section manifest the influence of the changeable vacuum field. We analyze an example for low-energy scattering of the influence of the changeable vacuum field upon the scattering cross section.

1. INTRODUCTION

One of the main issues in the general theory of relativity is the problem of zero-point or vacuum energy. The absolute scale of energy is of extreme importance in this theory, where the actual value of the energy-momentum tensor determines and the vacuum energy creates the curvature of space. In QED and QCD, the vacuum energy is usually dispensed with on the grounds that only differences in energy are measurable and the scale of energy may be made to start wherever we wish. The presence of vacuum energy can be established in physics through many other phenomena. Perhaps one of the most striking is the failure of liquid ^{4}He to solidify at normal pressures as the temperature is reduced toward absolute zero (Finkelburg, 1964). Bose condensation predicts that all helium atoms can fall into the same lowest energy state. However, the zero-point vibrations of this light atom are sufficient to overcome the attractions of the very small van der Waals forces, so that no crystal lattice is formed until considerable pressure is applied. The force on uncharged conducting parallel plates due to quantum electrodynamic

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effects was evaluated first by Casimir (1948) on the basis of zero-point energy arguments. The first experimental verification of the Casimir effect was performed by Spaarnay (1958). Fierz (1960) showed that under normal conditions the thermal radiation could be neglected. When electromagnetic fields are confined by ideal perfectly conducting plates or a cylinder, the Casimir effect yields an attractive force and a sphere gives rise to a repulsive force (Boyer, 1968).

Considering that the effect of the vacuum field may be observed in numerous phenomena, we decided to explore its impact on reactions between elementary particles. For the first time this paper presents the calculation of S-matrix and scattering cross section for the scattering of a photon on an electron located between two uncharged conducting parallel plates moving away from one another at a constant velocity. The electromagnetic vacuum field which exists between the plates interacts with the electron. The S-matrix and scattering cross-section calculations show that the vacuum field interacts with the Dirac field.

2. THE PHYSICAL SYSTEM

This paper presents the derivation of the propagator, S-matrix, and scattering cross section for scattering photons on electrons. The relevant reaction is illustrated in Fig. 1.

The electron is located between two uncharged conducting plates (Fig. 2). We assume that the dimensions of the plates L_x and L_y approach infinity, whereas the distance L_z between the two plates can be chosen. In our calculations we assume that the initial distance between the plates $L_z = 0.1-1$ μm at the moment when both plates start to move. This was the distance between the plates during the first successful experimental verification of the Casimir effect performed by Spaarnay. We further assume that at the moment $x_0 = 0$ the

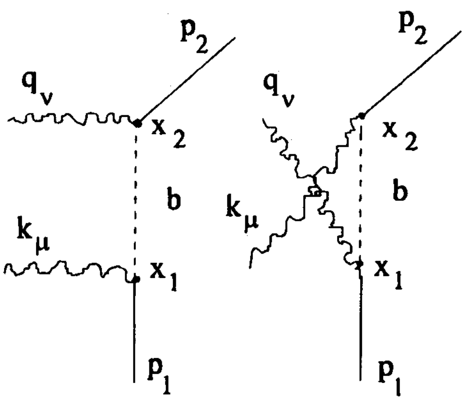

Fig. 1. Reaction diagrams.

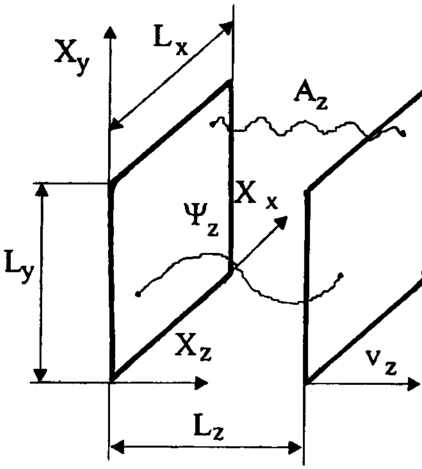

Fig. 2. Location of electron and photon fields.

plates start moving away from one another along the direction of the x_z axis at a constant velocity v_z. We did not make the computation for an accelerated movement of the plates; an accelerated neutral conducting plate emits photons (Unruh, 1976), which greatly complicates the calculation. We assume Minkowski space between the plates. Along the directions x_x and x_y the field may spread from $-\infty$ to $+\infty$, whereas along the direction x_z it is confined by the plates. All field variables are periodic functions of the space between the two plates.

The eigenvalues of operators for the photon and electron fields must equal zero on the plates. The field operators are also functions of the distance between the plates L_z and the velocity v_z of the movement of the plates. We introduce $x_z = v_z x_0$ for the x_z component. The propagator vector for the photon is

$$\mathbf{k} = \left(\frac{2\pi}{L_x} n_x, \frac{2\pi}{L_y} n_y, \frac{2\pi}{L_z} n_z\right)$$

The four-dimensional coordinates of the point are x_μ, where $\mathbf{x}_i$ represents the space coordinates and x_0 the temporal component. The relevant photon field operator is

$$\begin{aligned} A_\mu(x) &= \left(\frac{1}{2k_0 V}\right)^{1/2} \sum_{k_i} [a e_\mu^{(\lambda)} \sin(k_z v_z x_0) e^{ik_T x_T} \\ &\quad + a^+ e_\mu^{(\lambda)} \sin(k_z v_z x_0) e^{-ik_T x_T}] \qquad (1) \\ A_\nu(x) &= \left(\frac{1}{2q_0 V}\right)^{1/2} \sum_{q_i} [a e_\nu^{(\lambda)} \sin(q_z v_z x_0) e^{iq_T x_T} \\ &\quad + a^+ e_\nu^{(\lambda)} \sin(q_z v_z x_0) e^{-iq_T x_T}] \end{aligned}$$

In the field operators and in the S-matrix, x_z, P_z, q_z, k_z, and v_z are parallel to the x_z axis, whereas q_T, p_T, k_T, and x_T are the remaining four vector components. We used the units $\hbar = c = 1$.

The relevant operator for the electron field is

$$\Psi(x) = V^{-1/2} \sum_{p_i} \left[\sum_{r=1,2} a^{(r)} u^{(r)} \sin(p_z v_z x_0) e^{i p_T x_T} + \sum_{r=1,2} a^{+(r)} u^{(r)} \sin(p_z v_z x_0) e^{-i p_T x_T} \right] \tag{2}$$

$$\overline{\Psi}(x) = V^{-1/2} \sum_{p_i} \left[\sum_{r=1,2} a^{+(r)} \overline{u}^{(r)} \sin(p_z v_z x_0) e^{-i p_T x_T} + \sum_{r=1,2} a^{(r)} \overline{u}^{(r)} \sin(p_z v_z x_0) e^{i p_T x_T} \right]$$

In the field operators, a and a^+ represent the annihilation and creation operators, u are the spinors, and γ_ν, γ_μ the Dirac matrices. The following relations must be fulfilled:

$$\overline{u} = u^* \gamma_4, \qquad \tfrac{1}{2}(\gamma_\mu \gamma_\nu + \gamma_\nu \gamma_\mu) = \vartheta_{\mu\nu} \tag{3}$$

$$\{a_\alpha(p_i), a_\beta^+(p_i')\} = \delta_{\alpha\beta} \delta_{p_i, p_i'}, \qquad \{a_\alpha(p_i), a_\beta(p_i')\} = \{a_\alpha^+(p_i), a_\beta^+(p_i')\} = 0 \tag{4}$$

$$\{\Psi_\alpha(x_1), \overline{\Psi}_\beta(x_2)\} = i S_{\alpha\beta}(x_1 - x_2) \tag{5}$$

For the calculation of the Feynmann propagator for the electron we use equation (5), which in the interaction representation takes the form

$$S_F = \frac{i}{4} \lim_{\epsilon \to 0} \frac{1}{(2\pi)^3} \iiint d^3 b \sum_{b_z} e^{ib(x_1 - x_2)} \frac{i\gamma b - m}{b^2 + m^2 - i\epsilon} \tag{6}$$

In our case the propagator is also a function of velocity, as we used the relation $x_z = v_z x_0$. Due to the final distance between the plates we could not use the standard equation

$$\sum_{p_i} \to \frac{V}{(2\pi)^3} \iiint d^3 b$$

We carried out the summation along the direction of the z axis and this is where our calculation of the Feynmann propagator differs from the traditional ones. We used

$$\sum_{p_i} \rightarrow \frac{V}{(2\pi)^3} \iint \sum_{b_z} d^2 b$$

3. S-MATRIX CALCULATION

Figure 1 shows the vertex diagrams for the reaction. The relevant S-matrix of the lowest order, which is identical to the S-matrix for Compton scattering, is

$$\begin{aligned} &\langle p_2, q | S | p_1, k\rangle \\ &= e_0^2 \int d^3 x_{1T} \int_0^{v_z x_{10}} dx_{1z} \int d^3 x_{2T} \int_0^{v_z x_{20}} dx_{2z} \\ &\times \langle p_2 | \overline{\Psi}(x_1) | 0\rangle \gamma_\mu S_F(x_1 - x_2)\gamma_\nu \\ &\times : \langle 0 | \Psi(x_2) | p_1\rangle : \langle q | A_\mu(x_1) A_\nu(x_2) | k\rangle : \end{aligned} \tag{7}$$

After the introduction of field operators for photons (1) and electrons (2) as well as the propagator (6), the S-matrix takes the form

$$\begin{aligned} &\langle p_2, q | S | p_1, k\rangle \\ &= e^2 \int d^3 x_{1T} \int_0^{v_z x_{10}} dx_{1z} \int d^3 x_{2T} \int_0^{v_z x_{20}} dx_{2z} \\ &\times V^{-1/2} \overline{u}^{(r)} \sin(p_2 v_z x_{10}) e^{-p_{2T} x_{1T}} \\ &\times \gamma_\mu \frac{i}{4} \frac{1}{(2\pi)^3} \iiint d^3 b \sum_{b_z} e^{ib(x_1 - x_2)} \frac{i\gamma b - m}{b^2 + m^2 - i\epsilon} \gamma_\nu \\ &\times V^{-1/2} u^{(r)} \sin(p_{1z} v_z x_{20}) e^{ip_{1T} x_{2T}} \left(\frac{1}{4 q_0 k_0 V^2}\right)^{1/2} \\ &\times [e_\nu^{(\lambda_1)} \sin(k_z v_z x_{20}) e^{ik_T x_{2T}} e_\mu^{(\lambda_2)} \sin(q_z v_z x_{10}) e^{-iq_T x_{1T}} \\ &+ e_\nu^{(\lambda_1)} \sin(k_z v_z x_{10}) e^{ik_T x_{1T}} e_\nu^{(\lambda_2)} \sin(q_z v_z x_{20}) e^{-iq_T x_{2T}}] \end{aligned} \tag{8}$$

Due to the movement and the final distance between the plates the calculation of the S-matrix becomes complicated. We were unable to carry out the usual integration from $-\infty$ to $+\infty$ along the direction of x_z. Thus, we decided to integrate from 0 to $L_z = v_z x_0$. Along the direction of the remaining three components we integrated from $-\infty$ to $+\infty$. Another difficulty in the calculation of the S-matrix is that it is impossible to carry out the standard integration over the momentum along the direction of the x_z axis. Then, after introducing

$b_z = (2\pi/L_z)n_z$ in the S-matrix (8), summing over n_z from 1 to ∞, and carrying out the standard integration on the other components, we find the S-matrix as

$$\langle p_2, q | S | p_1, k\rangle$$

$$= \frac{ie^2}{(4k_0q_0)^{1/2}}\frac{1}{32\cdot 4}\bar{u}_2(p_2)\gamma_\mu e_\mu^{(\lambda_2)}\left\{\left[\frac{i\gamma_\mu(p_1+k)-m}{(v_0+1)^2[(p_1+k)^2+m^2]}\right.\right.$$

$$+\left(\frac{i\gamma_z}{2(v_0+1)^2[(p_{1T}+k_T)^2+m^2]}-\frac{i\gamma_T(p_{1T}+k_T)-m}{2i(v_0+1)^2[(p_{1T}+k_T)^2+m^2]}\right)$$

$$\left.\times\left(\frac{v_0^2\pi}{2L_z}\coth[(p_{1T}+k_T)^2+m^2]L_z-\frac{\pi}{L_z^2[(p_{1T}+k_T)^2+m^2]}\right)\right]$$

$$+\left[\frac{i\gamma_\mu(p_1-q)-m}{(v_0+1)^2[(p_1-q)^2+m^2]}\right.$$

$$+\left(\frac{i\gamma_z}{2(v_0+1)^2[(p_{1T}-q_T)^2+m^2]}-\frac{i\gamma_T(p_{1T}-q_T)-m}{2i(v_0+1)^2[(p_{1T}-q_T)^2+m^2]}\right)$$

$$\left.\left.\times\left(\frac{v_0^2\pi}{2L_z}\coth[(p_{1T}-q_T)^2+m^2]L_z-\frac{\pi}{L_z^2[(p_{1T}-q_T)^2+m^2]}\right)\right]\right\}$$

$$\times e_\nu^{(\lambda_1)}\gamma_\nu u(p_1)(2\pi^3)\delta^3(p_{1T}+k_T-p_{2T}-q_T)\,\delta(n_{p_{1z}}+n_{k_z}-n_{p_{2z}}-n_{q_z})$$

$$= F(2\pi)^3\delta^3(p_{1T}+k_T-p_{2T}-q_T)\,\delta(n_{p_{1z}}+n_{k_z}-n_{p_{2z}}-n_{q_z}) \tag{9}$$

If the two plates are infinitely distant from one another and their velocity is zero, the S-matrix turns into the standard form for Compton scattering. When there is a final distance between the plates and their velocity is zero, only the vacuum field affects the S-matrix correction.

4. SCATTERING CROSS SECTION

In the calculation of the scattering cross section we assume the laboratory system and use the following equation (Gupta, 1977) to calculate the cross section:

$$\sigma = \frac{1}{(2\pi)^2|p_{1i}/p_{10}-k_i/k_0|}\int d\Omega\,\frac{|p_{2i}|p_0}{\partial(p_{10}+q_0)/\partial p_{10}}\overline{\sum}|F(p_2, q; p_1, k)|^2$$

$$\overline{\sum} = \overline{\sum_{\text{spin}}}\,\overline{\sum_{\text{pol}}} \tag{10}$$

$\overline{\Sigma}_{\text{spin}}$ denotes the average over the initial spin states and summation over the final spin states of the electron. The statement holds in cases when we are not interested in the spin state of the particle during the reaction. $\overline{\Sigma}_{\text{pol}}$ denotes a similar average and summation over the polarization of photons. To simplify the S-matrix we used the following relations:

$$p_{1i} = 0, \qquad p_{10} = m$$

$$k_i = p_{2i} + q_i, \qquad p_1^2 = p_2^2 = -m^2, \qquad k^2 = q^2 = 0$$

$$(p_1 + k)^2 + m^2 = 2p_1k, \qquad (p_1 - q)^2 + m^2 = -2p_1q$$

$$(p_{1T} + k_T)^2 + m^2 = 2p_{1T}k_T, \qquad (p_{1T} - q_T)^2 + m^2 = -2p_{1T}q_T$$

$$(ip_1\gamma - m)(e_i^{(\lambda_2)}\gamma_i)u(p_1) = 0, \qquad (ip_1\gamma - m)(e_i^{(\lambda_1)}\gamma_i)u(p_1) = 0$$

After the simplification of the S-matrix (9), we calculate F^2 and introduce it into equation (10). The resulting scattering cross section for polarized light is

$$\begin{aligned}\sigma_{\text{pol}} = \frac{1}{4} r_0^2 \int d\Omega \Bigg\{ &\frac{q_0^2}{k_0^2}\left[\frac{k_0}{q_0} + \frac{q_0}{k_0} - 2 + 4(ee')^2\right] \\ &- \frac{q_0^2 p_{20}}{k_0^2 \cdot 128}\Bigg[\frac{v_0^4}{4L_z^4(v_0+1)^2(v_0+1)^2k_0^4}(ee')^2\frac{\pi}{4}\coth^2(-2mk_0L_z) \\ &+ \frac{\pi^2(ee')^2}{16L_z^4(v_0+1)^2(v_0+1)^2k_0^4} \\ &+ \frac{v_0^4}{4L_z^4(v_0+1)^2(v_0+1)^2q_0^4}(ee')^2\frac{\pi}{4}\coth^2(2mq_0L_z) \\ &+ \frac{\pi^2(ee')^2}{16L_z^4(v_0+1)^2(v_0+1)^2q_0^4}\Bigg]\Bigg\}\end{aligned} \tag{11}$$

When $v_z \to 0$ and $L_z \to \infty$ the scattering cross section (11) is identical to the cross section for polarized photons calculated by Klein and Nishina.

The scattering cross section is calculated also for the case when $k_0 = q_0$ and $k_0 << q_0$. In equation (11) we insert

$$k_0 = \frac{2\pi}{\lambda}, \qquad q_0 = \frac{2\pi}{\lambda'}, \qquad \lambda = \lambda'$$

A short calculation gives

$$\begin{aligned}\sigma_{\text{pol}} = \frac{1}{4}\int d\Omega\, r_0^2(ee')^2\Bigg[1 - \frac{1}{128}\Bigg(&\frac{v_0^4\lambda^4}{4(v_0+1)^2(v_0+1)^2L_z^4} \\ &+ \frac{\pi^2\lambda^4}{16(v_0+1)^2(v_0+1)^2L_z^4}\Bigg)\Bigg]\end{aligned} \tag{12}$$

In equation (12) we must consider the condition of the photon wavelength being $\lambda \leq L_z$. We use the known relation for the summation over the polarized states

$$\overline{\sum_{\text{pol}}} (ee')^2 = \frac{1}{2}(1 + \cos^2\Theta) \tag{13}$$

The scattering cross section for unpolarized low-energy photons is

$$\sigma = r_0^2 \int d\Omega \left(1 - \frac{1}{2}\sin^2\Theta\right)\left[1 - \frac{1}{128}\left(\frac{v_0^4\lambda^4}{4L_z^4(v_0 + 1)^2(v_0 + 1)^2} + \frac{\pi^2\lambda^4}{16L_z^4(v_0 + 1)^2(v_0 + 1)^2}\right)\right] \tag{14}$$

5. CONCLUSION

The terms in equations (11), (12), and (14) which contain the distance between the plates L_z and their velocity v_z are correction terms and represent the effect of the vacuum field and the velocity of the plate movement on the scattering cross section. When $L_z = \infty$ and $v_z = 0$, the correction terms are zero and equation (11) turns into the well-known Klein–Nishina formula, whereas equations (12) and (14) are converted into the standard formulas for the scattering cross sections of polarized and unpolarized photons.

When $v_z = 0$ and $0 < L_z < \infty$, the last term on the right side of equations (12) and (14) represents the corrections to these equations. This term represents the effect of the vacuum field on the scattering cross section when the plates are at rest. The larger the distance between the plates and the shorter the wavelength of the photon, the smaller the effect of the vacuum field. The correction term depends on L_z. The dependence of the attraction force and the vacuum energy between the plates on L_z was also established by Casimir (1948).

As the plates move away from one another at a constant velocity v_z, the correction of the scattering cross section is represented by the last two terms on the right side of equations (12) and (14). The purpose of the S-matrix and scattering cross-section calculation was to establish, on a concrete example, the effect of the varying vacuum field on the scattering of photons on an electron. All previous calculations of S-matrix and scattering cross sections for the reactions between the basic particles have not considered the effect of the vacuum field and assumed that the fields could propagate from $-\infty$ to $+\infty$.

Boyer (1968) carried out the calculation of the vacuum energy for a universe of any diameter. We believe that the vacuum field would have a substantial effect on the reactions between the particles in the early stage of the creation of the universe when its diameter was very small. This effect would also be significant when the diameter of the universe is chosen to be very close to the Planck length.

The exploration of the effect of the vacuum field on the reactions when the diameter of the universe is close to the Planck length will be the subject of our future research.

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